On Modeling the Coexistence of 802.11 and 802.15.4 Networks for Performance Tuning

Wei Zhang, *Student Member, IEEE*, Mahima A. Suresh, *Student Member, IEEE*, Radu Stoleru, *Member, IEEE*, and Harsha Chenji, *Student Member, IEEE*

***Abstract*—The explosion in the number of 802.11 and 802.15.4 deployments is exacerbating the coexistence problem, which has been reported in the literature to cause significant performance degradation in co-located networks employing the two different wireless standards. The wireless coexistence problem has, thus far, been studied primarily using hardware, due to the lack of analytical results and good wireless coexistence simulators. This paper presents the first analytical model for coexisting 802.11 and 802.15.4 networks. We derive analytically, using Markov chains, the normalized saturation throughput under coexistence. Additionally, we propose a performance tuning method that ensures QoS and a distributed Nash-equilibrium-based method that ensures fairness. We validate our model and the tuning methods using a coexistence simulator previously developed and presented by the authors. We demonstrate that our model has a low average error smaller than 10%.**

***Index Terms*—Wireless networks, IEEE 802.11 Standards, Zigbee, performance analysis, Markov processes, quality of service.**

I. Introduction

In recent years, we have witnessed an exponential growth in the usage of WiFi (i.e., IEEE 802.11) networks and wireless sensor (i.e., IEEE 802.15.4) networks. The ubiquitous nature of these wireless network deployments (which share the 2.4 GHz ISM unlicensed frequency band) combined with the significant throughput degradation reported in the literature to occur in coexisting 802.11/802.15.4 deployments [1], [2] emphasize the urgent need for a rigorous study of the coexistence of these two different wireless protocols.

There are two possible coexistence scenarios. In the first scenario, which we call "asymmetric coexistence," a wireless node can detect transmissions from a second wireless device, while its transmissions cannot be detected by the second device. This scenario can occur either due to different communication ranges (i.e., asymmetric) or because of differences in PHY layer modulations. For example, an older 802.11b, radio which does not employ energy-based OFDM [3] cannot detect 802.15.4 transmissions. In the second scenario, which we call "symmetric coexistence," both wireless devices can detect the transmissions of each other. With the advent of newer standards, like the energy modulation based 802.11ac, as well as long range 802.15.4 radios (e.g., the Freescale ZigBee range extender), this symmetric coexistence scenario is expected to become more ubiquitous in the future. Thus, in this article, we focus on the symmetric coexistence of 802.11 and 802.15.4 wireless protocols.

To address the aforementioned performance degradation under coexistence, research has focused on channel allocation techniques [4]–[6], which work for wireless networks under both symmetric and asymmetric coexistence. This techniques, however, are becoming less efficient due to the exponential growth in deployments of WiFi and other technologies using the ISM band (e.g., Bluetooth, microwave). Other techniques, such as transmissions scheduling which are based on features of the protocols or on knowledge about traffic patterns, have also been proposed [7]–[11]. These have shown promising results to mitigate the performance degradation for wireless networks with asymmetric coexistence only. The symmetric coexistence of 802.11 and 802.15.4 wireless standards (both CSMA based, but with different backoff mechanisms, time slots and protocol parameters) has largely remained unexplored. More precisely, the two protocols, when in symmetric coexistence, exhibit QoS and fairness problems. It remains a research challenge to develop a rigorous coexistence model and analysis which can be used for deriving performance metrics such as throughput and delay.

To address this challenge, in this article we present *the first analytical model* for 802.11 (as 802.11 DCF) and 802.15.4 (as BoX-MAC [12]) for networks with symmetric coexistence. Our analysis builds on extended and improved Markov Chain models for 802.11 DCF [13] and ZigBee [14] and it provides a fast and scalable way to predict saturation throughput. It is paramount to note that, due to the differences between these two protocols, the modeling of such coexistence is far more difficult than the modeling of collocated devices employing a single protocol, which was extensively studied. Additionally, for demonstrating the usefulness of our analytical model, we present two contention window size tuning methods that address the aforementioned QoS and fairness problems under symmetric coexistence. We note here that for symmetric coexistence all devices are within one hop, i.e., single cell. We leave for future work more complex scenarios, such as multiple hop

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W. Zhang, M. A. Suresh, and R. Stoleru are with the Department of Computer Science and Engineering, Texas A&M University, College Station, TX 77843 USA (e-mail: wzhang@cse.tamu.edu; agumbe@cse.tamu.edu; stoleru@cse.tamu.edu).

H. Chenji was with the Department of Computer Science and Engineering, Texas A&M University, College Station, TX 77843 USA. He is now with the University of Texas at Dallas, Richardson, TX 75083-0688 USA (e-mail: cjh@cse.tamu.edu).

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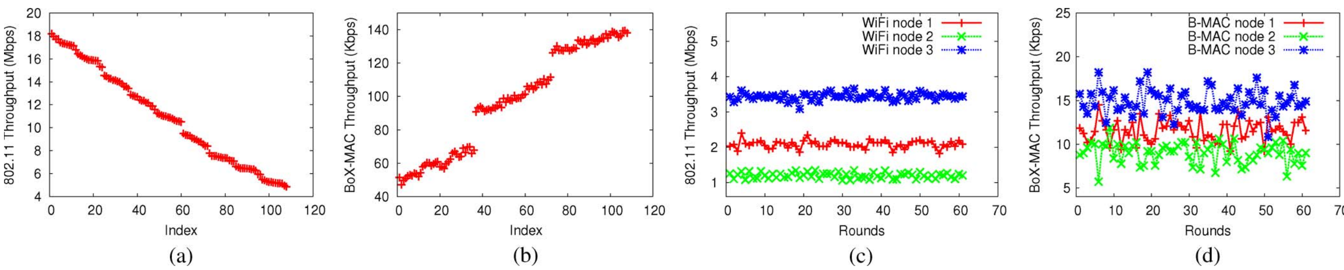

Fig. 1. Demonstrating the (a)–(b) QoS problem and (c)–(d) fairness problem in coexisting WiFi and BoX-MAC networks.

networks, where hidden and exposed terminal problems might be present.

The contributions of this article are as follows: 1) it presents the first analysis for saturation throughput for symmetric coexistence of IEEE 802.11 DCF and BoX-MAC; 2) it proposes a new Markov Chain based channel model that can accurately predict channel busy probabilities; 3) it presents two contention window tuning methods, one centralized and one distributed, that can achieve QoS and fairness, respectively; and 4) it demonstrates the accuracy of the model and the effectiveness of our tuning methods through extensive comparisons with a first-of-its-kind Monte Carlo based simulator for symmetric coexistence [15].

The rest of this article is organized as follows. Section II motivates this article by demonstrating the impact of symmetric coexistence on throughput, and presents the state of art for modeling wireless coexistence. Section III presents Markov Chain models for 802.11 and BoX-MAC, and their steady state analysis, and a Markov Chain based channel model. These are used to derive saturation throughput for coexisting 802.11 and BoX-MAC networks. Section IV proposes two contention window tuning methods. Section V presents extensive performance evaluations while Section VI concludes this article and presents ideas for future work.

II. Motivation and State of Art

First, we motivate the coexistence problem by showing how throughput is affected in a symmetric coexistence scenario consisting of WiFi and BoX-MAC devices. Next, we thoroughly review state of art and place our work in context.

A. Motivation

Since Contention Window size (CW) is a well-known critical parameter that affects throughput and fairness [16]–[19], as studied in typical wireless networks employing single MAC protocols, we were curious to investigate how CW sizes of different, coexisting MAC protocols affect throughput and fairness of networks with symmetric coexistence.

For this, we have used a Monte Carlo based simulator [15] specifically designed to handle coexisting MAC protocols. The simulator does not account for the duty cycling feature of BoX-MAC, since we are interested in saturated traffic. We simulated 5 WiFi and 10 BoX-MAC devices, all within one hop, in two sets of experiments. In the first set of experiments we ran multiple simulations, each with a different combination of contention window sizes, CW_W for WiFi and CW_B for BoX-MAC. Importantly, in each simulation, the devices of the same type employ the same contention window size. In the second set of experiments we ran multiple simulations and allowed devices of the same type to choose their contention window size (CW_W for WiFi and CW_B for BoX-MAC) freely. The traffic was saturated in both sets of experiments.

The results for the first sets of experiments are presented in Fig. 1(a) and (b). As shown, the throughput of WiFi and BoX-MAC are strictly inversely proportional. This motivates us to provide *Quality of Service (QoS)* where a network administrator can decide an operating point. The results for the second set of experiments are presented in Fig. 1(c) and (d) which show that if devices of the same type are allowed to use different CW, they may experience different throughput, thus experiencing the *fairness problem*. When nodes arbitrarily set their CWs to benefit themselves, channel sharing is unfair.

The state of art research only focused on the effects of CW size on the performance of a single MAC protocol. In a symmetric coexistence scenario, different protocols (WiFi and BoX-MAC) have different CW sizes. Intuitively, CW size will affect the throughput and fairness of both protocols, but the extent to which they will be affected, is not known.

B. State of Art

Modeling CSMA protocols using Markov Chain has received significant attention. The seminal paper by Bianchi [20] is the first to describe the binary exponential backoff mechanism of 802.11 DCF as a 2-D Discrete Time Markov Chain. To address inaccuracies in that model (due to absence of retransmission limits and the backoff counter freezing) variants were proposed [21], [22]. Most recently, Felemban *et al.* [13] proposed a Markov Chain based channel model to estimate the freezing probability, and demonstrated significant improvement in modeling accuracy. Researchers have also developed similar models for other protocols. Pollin *et al.* [14] proposed an accurate model for IEEE 802.15.4, for both saturated and non-saturated traffic. It is paramount to note that these models are all for single MAC protocols, i.e., non coexistence. Notably, researchers have attempted to build models for coexisting networks [23], [24], but all these employ variants of the same 802.11 MAC protocol, e.g., 802.11b and 802.11g.

Several approaches have been proposed to support coexistence of 802.11 and 802.15.4 devices. The most popular, yet simple, approach is to assign orthogonal channels to WiFi and WSN devices [4]–[6]. Certainly, this technique is applicable to wireless networks with both symmetric and asymmetric coexistence. This solution, however, becomes ineffective in highly dense, pervasive, deployments of devices operating in the ISM band. Other approaches for handling wireless coexistence were based on the characteristics of WiFi and WSN signals [7]–[11]. These approaches, with promising results, are only for asymmetric coexistence scenario (e.g., WiFi's impact on ZigBee) and could not be generalized to other situations.

For performance tuning of wireless networks, methods that employ CW size adaptation were proposed [16]–[19]. These methods are either centralized or distributed. They typically propose models for throughput, delay and fairness, then make estimates for collision probabilities and number of devices. Finally, by solving various optimization problems, optimal CW sizes are derived. However, it is often very difficult to accurately estimate the number of devices in the distributed CW size adaptation methods. To address this problem, recently, game theoretic solutions were proposed [25], [26]. Essentially, these solutions optimize a payoff function defined as the difference between a utility function (e.g., throughput) and a price function (e.g., collision rate). Since each device in a game theoretic solution only observes the price, and needs not know the number of devices, the solution is distributed inherently. However, all existing performance tuning methods employing game theoretic approaches are for devices of the same type, and cannot be employed in coexistence scenarios (symmetric or asymmetric).

Recently, the first analytical model for coexistence was proposed [15]. Using existing Markov Chain models for 802.15.4 [14] and 802.11 [20], a combined model was formulated, and mathematical expressions for aggregate throughput were derived. The model was evaluated using a newly built Monte Carlo based coexistence simulator, the first of its kind. This article improves upon [15] in three ways. First, the channel busy probabilities are predicted more accurately by deriving and using a new *Markov Chain-based Channel model*—leading to better throughput estimates. Second, we propose two contention window tuning methods to achieve QoS and fairness in wireless networks with symmetric coexistence. Finally, we evaluate our new model using a larger network, employing 20 802.11 and 30 BoX-MAC devices.

III. Models and Analysis for Coexistence of WiFi and BoX-MAC

In this section, we present the mathematical analysis for the coexistence of 802.11 DCF and BoX-MAC. We assume that the traffic is saturated and that the devices are within communication range of each other (i.e., symmetric coexistence). To the best of our knowledge, this is the first analysis for coexistence of these two classes of devices. For analysis, we use independent analytical models for the two MAC protocols, followed by their steady state analysis. A novel Markov Chain based *channel model* estimates the channel busy probability. These enable us to compute saturation throughput for the symmetric coexistence model.

Both 802.11 DCF and BoX-MAC are modeled as Markov Chains. By the Markov property, state transition probabilities are dependent only on the most recent states. We model a state transition at the end of a time slot whose size is dependent on the protocol. It is hard to analyze coexistence if two devices have different time slot sizes. In this case, BoX-MAC has a time slot that is 3 times that of 802.11. To account for the difference in slot sizes, while maintaining the Markovian property, we add two dummy states to each BoX-MAC state, each with transition probability 1, as explained below. Therefore, each state in the Markov Chain corresponds to one third of a BoX-MAC time slot, i.e., each BoX-MAC slot is divided in three equal time slots.

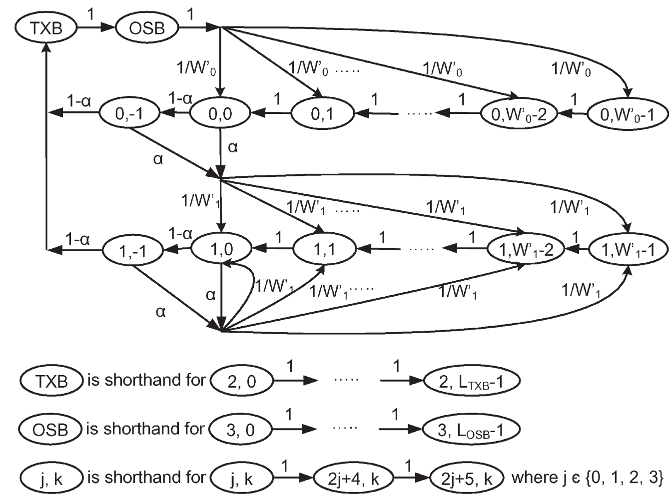

Fig. 2. The Markov Chain describing the BoX-MAC protocol.

A. Markov Chain Model for BoX-MAC

The BoX-MAC protocol is a simplified version of the IEEE 802.15.4 protocol, which makes it tenable to mathematical analysis. Similar to the approach in [14], we model the BoX-MAC protocol as a Markov Chain (shown in Fig. 2). Let $s(t)$ and $c(t)$ be the stochastic processes representing the high level *stage* and *counter*, respectively. The process where each state is represented by $(s(t), c(t))$ can be modeled as a Markov Chain. The high level stages of a BoX-MAC node are: backoff with initial contention window $CW_{\rm init}$, backoff with congested contention window $CW_{\rm cong}$, transmission and operating system delays. The counter process accounts for the number of time slots corresponding to each high level stage.

The states (j, k), $(2j+4, k)$, and $(2j+5, k)$, where $j \in \{0, 1, 2, 3\}$, represent one third of a BoX-MAC slot. A node entering state (j, k) transitions to state $(2j+4, k)$, and then to $(2j+5, k)$ with probability 1 at the end of each time slot, thereby accounting for an entire BoX-MAC slot. We denote by W'_j the current contention window size, where $j \in \{0, 1\}$. When $j = 0$ the stage $s(t) \in \{0, 4, 5\}$ and the node is in backoff stage with a contention window size $CW_{\rm init}$ ($W'_0 = CW_{\rm init}$). When $j = 1$ the stage $s(t) \in \{1, 6, 7\}$ and $W'_1 = CW_{\rm cong}$. The backoff delay is represented by states (j, k), where $j \in \{0, 1\}$ and $k \in \{-1, 0, 1, \ldots, W'_j - 1\}$. The transition probabilities are assumed to be independent. As described by the CSMA/CA mechanism, a device starts from state $(0, k)$, where k is a random number between 0 and $CW_{\rm init} - 1$. In states $(j, 0)$ and $(j, -1)$, a channel assessment (CCA) is performed. We note that any channel sensing state, such as (0,0) or (1, −1), is modeled by a process denoted as *SENS_B* and will be used in the channel model (Section III-D). If the channel is sensed busy (with probability α), the device transitions to state $(1, k)$ where k is a random number between 0 and $CW_{\text{cong}} - 1$. If the channel is sensed idle in both states $(j, 0)$ and $(j, -1)$, the packet is transmitted.

Transmission states are represented as TXB; specifically, $s(t) \in \{2, 8, 9\}$ and $c(t) \in \{0, 1, \ldots, L_{\text{TXB}} - 1\}$, where L_{TXB} is the duration of a BoX-MAC transmission, and a function of the packet size and transmission bandwidth. Before sending a packet, the device experiences delay from the operating system. This is represented as state OSB; specifically, $s(t) \in \{3, 10, 11\}$ and $c(t) \in \{0, 1, \ldots, L_{\text{OSB}} - 1\}$, where L_{OSB} is the operating system delay, obtained experimentally. For ease of understanding the equations that follow, we extend the notation W'_j such that it denotes L_{TXB} and L_{OSB}, i.e., $W'_0 = CW_{\text{init}}$, $W'_1 = CW_{\text{cong}}$, $W'_2 = L_{\text{TXB}}$ and $W'_3 = L_{\text{OSB}}$. Thus, the valid values for state j in (j, k) are $\{0, 1, 2, 3\}$.

The single step transition probabilities, as defined by the Markov Chain for BoX-MAC, are:

$$\begin{aligned}
\Pr[2j+4, k|j, k] &= 1,\ k = -1, 0, \ldots, W'_j - 1\\
\Pr[2j+5, k|2j+4, k] &= 1,\ k = -1, 0, \ldots, W'_j - 1\\
\Pr[j, k-1|2j+5, k] &= 1,\ j = 0, 1;\ k = 1, \ldots, W'_j - 1\\
\Pr[j, -1|2j+5, 0] &= 1 - \alpha,\ j = 0, 1\\
\Pr[1, k|2j+5, 0] &= \alpha/W'_1,\ j = 0, 1;\ k = 0, \ldots, W'_1 - 1\\
\Pr[1, k|2j+5, -1] &= \alpha/W'_1,\ j = 0, 1;\ k = 0, \ldots, W'_1 - 1\\
\Pr[2, 0|2j+5, -1] &= 1 - \alpha,\ j = 0, 1\\
\Pr[j, k|2j+5, k-1] &= 1,\ j = 2, 3;\ k = 1, \ldots, W'_j\\
\Pr[3, 0|9, L_{TXB} - 1] &= 1\\
\Pr[0, k|11, L_{OSB} - 1] &= 1/W'_0,\ k = 0, \ldots, W'_0
\end{aligned}$$

where $j \in \{0, 1, 2, 3\}$ if not explicitly specified. The coexistence of BoX-MAC with other types of devices can be modeled by computing α, i.e., the probability that the channel is busy during a given time slot. The probabilities of sensing a busy channel during the first and second CCA are typically correlated [14]. However, we approximate them as independent events. We validate experimentally that this inaccuracy is tolerable.

B. Markov Chain Model for IEEE 802.11

The Markov Chain model that we propose for 802.11 DCF is depicted in Fig. 3. We extend the Bianchi model [20] to include backoff freezing and we adopt ideas for accurate modeling, as in Felemban and Ekici [13]. Our model uses two parameters $b(t)$ and $r(t)$ for the high level *stage* (e.g., backoff, transmission, etc.) and *counter*, respectively. The counter is used as an indicator for the number of time slots in each stage. Each state is represented as $(b(t), r(t))$. This model does not account for inter-frame spacings when the channel is sensed busy in a backoff state.

The stages where $b(t) \geq 0$ correspond to backoffs. When an 802.11 device attempts to transmit a packet, it starts at state

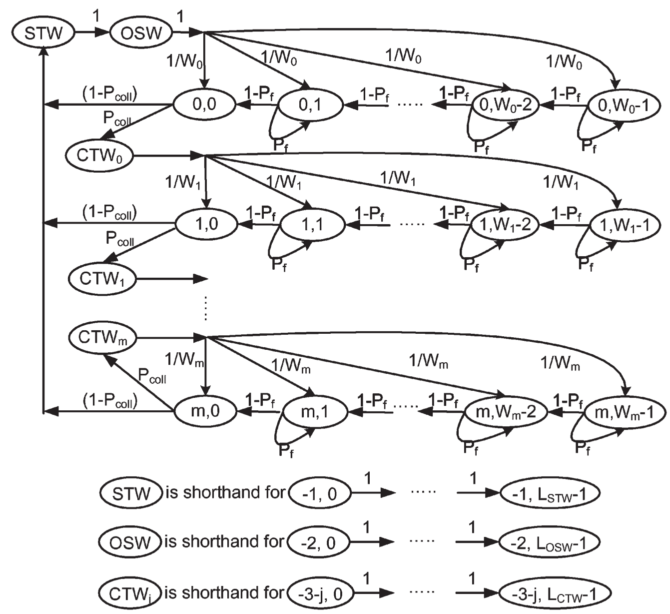

Fig. 3. The Markov Chain describing 802.11 DCF.

$(0, k)$, where k is a random number between 0 and $CW_{\min}$, ($CW_{\min}$ is the minimal contention window). The channel is sensed in each time slot (state). Similar to BoX-MAC, any channel sensing state for 802.11 is modeled by a process named $SENS_W$. If the channel is sensed busy (with probability P_f), the device remains in the same state, i.e., the backoff counter freezes. If the channel is free, the backoff counter is decremented. When the backoff counter reaches 0, if the channel is sensed idle, the 802.11 device transmits the packet and waits for an acknowledgement (ACK). If an ACK is not received (e.g., due to collision), the device tries to transmit the packet again, with a current contention window W_j, being in backoff stage j. The probability that a transmitted packet collides with others is P_{coll}. Collisions occurring after backoff stage j (i.e., $(j, 0)$), where $j = 1, 2, \ldots, m$, are represented by states CTW_j. In these states $b(t) = -3 - j$ and $r(t) = 0, 1, \ldots, L_{\text{CTW}} - 1$. When backoff stage m is reached, further retries are still made from the same backoff stage and have the maximum contention window size $CW_{\max}$. We note that $m = \log_2(CW_{\max}/CW_{\min})$.

When an ACK is received, i.e., the packet is successfully sent, the next packet transmission is attempted after experiencing some delay from the operating system, represented as state OSW, where $b(t) = -2$, and $r(t) = 0, 1, \ldots, L_{\text{OSW}} - 1$. A successful transmission is represented as state STW, where $b(t) = -1$, and $r(t) = 0, 1, \ldots, L_{\text{STW}} - 1$. Here, L_{STW} and L_{CTW} are functions of packet size, the available bandwidth and MAC protocol specific delays such as interframe spacing and ACK timeout. L_{OSW} is obtained from hardware experiments. Similar to the notation W'_j in BoX-MAC, we extend W_j such that it can denote L_{STW}, L_{CTW} and L_{OSW} of 802.11.

The single step transition probabilities, as defined by the Markov Chain for 802.11, are:

$$\begin{aligned}
\Pr[j, k-1|j, k] &= 1 - P_f\\
\Pr[j, k|j, k] &= P_f
\end{aligned}$$

TABLE I
EXPRESSIONS FOR THE LIMITING DISTRIBUTION OF 802.11 AND BOX-MAC MARKOV CHAINS

BoX-MAC $(k=0,\ldots,W'_j-1)$	**802.11 DCF** $(k=0,\ldots,W_j-1)$
$b'_{j,k}=\frac{W'_j-k}{W'_j}b'_{j,0}$, $j=0,1$	$b_{j,0}=b_{0,0}P^j_{coll}$, $j=0,\ldots,(m-1)$
$b'_{1,0}=\frac{b'_{0,0}x}{1-x}$, $j=0,1$	$b_{m,0}=b_{0,0}\frac{P^m_{coll}}{1-P_{coll}}$
$b'_{j,-1}=(1-\alpha)b'_{j,0}$, $j=0,1$	$b_{j,k}=\frac{1}{1-P_f}\frac{W_j-k}{W_j}b_{j,0}$, $j=0,\ldots,m$
$b'_{j,k}=(1-\alpha)^2(b'_{0,0}+b'_{1,0})$, $j=2,3$	$b_{-3-j,k}=b_{j,0}P_{coll}$, $j=0,\ldots,m$
$b'_{2j+4,k}=b'_{j,k}$, $j=0,1,2,3$	$b_{-1,k}=b_{0,0}$
$b'_{2j+5,k}=b'_{j,k}$, $j=0,1,2,3$	$b_{-2,k}=b_{0,0}$

$$
\begin{aligned}
\Pr[-3-j,0|j,0] &= P_{coll}\\
\Pr[j,k|-3-(j-1),L_{\mathrm{CTW}}] &= 1/W_j,\ j=1,\ldots,m\\
\Pr[m,k|-3-m,L_{\mathrm{CTW}}] &= 1/CW_{\max}\\
\Pr[-1,0|j,0] &= 1-P_{coll}\\
\Pr[-2,0|-1,L_{\mathrm{STW}}] &= 1\\
\Pr[0,k|-2,L_{\mathrm{OSW}}] &= 1/CW_{\min}\\
\Pr[j,k|j,k-1] &= 1,\ j=-1,-2,\ldots-3-m
\end{aligned}
$$

where $j=0,\ldots,m$ if not specified explicitly; $k=0,1,\ldots,W_j-1$, where $W_j=CW_{\min}2^j$ when $j=0,1,\ldots,m$; and $W_{-1}=L_{\mathrm{STW}}$, $W_{-2}=L_{\mathrm{OSW}}$, $W_l=L_{\mathrm{CTW}}$ when $l=-3,-4,\ldots,-3-m$.

In order to model the coexistence of WiFi with other types of devices, we need to compute the variables P_{coll} and P_f. These probabilities reflect the state of the channel.

C. Steady State Analysis

First we perform steady state analysis in order to obtain the stationary distributions for both BoX-MAC and 802.11 Markov chains, and their normalization conditions. These are then used to obtain the transmission probabilities and the conditional collision probability *under coexistence*.

Let $b'_{j,k}=\lim_{t\to\infty}\Pr\{s(t)=j,c(t)=k\}$ be the stationary distribution of the BoX-MAC Markov Chain and $b_{j,k}=\lim_{t\to\infty}\Pr\{b(t)=j,r(t)=k\}$ be the stationary distribution of the 802.11 Markov Chain. Expressions for all the terms in the limiting distribution of the Markov Chains are presented in Table I, where $x=(\alpha+(1-\alpha)\alpha)$, $W'_0=CW_{\mathrm{init}}$ and $W'_1=CW_{\mathrm{cong}}$.

The normalization condition is used for obtaining $b'_{0,0}$ and $b_{0,0}$ from the Markov Chains. The following equations are for BoX-MAC:

$$
\begin{aligned}
1 &= \sum_{j=0}^{1}\sum_{k=0}^{W'_j-1}3b'_{j,k}+\sum_{k=0}^{L_{\mathrm{TXB}}-1}3b'_{2,k}+\sum_{k=0}^{L_{\mathrm{OSB}}-1}3b'_{3,k}\\
b'_{0,0} &= \frac{1}{\left(3\frac{W'_0+1}{2}+3\frac{(W'_1+1)x}{2(1-x)}+3\frac{1-\alpha}{1-x}+3L_{\mathrm{TXB}}+3L_{\mathrm{OSB}}\right)}
\end{aligned} \tag{1}
$$

where $W'_0=CW_{\mathrm{init}}$ and $W'_1=CW_{\mathrm{cong}}$.

The following equations are for 802.11 DCF:

$$
\begin{aligned}
1 &= \sum_{j=0}^{m}\sum_{k=0}^{W_j-1}b_{j,k}+\sum_{j=0}^{m}\sum_{k=0}^{L_{\mathrm{CTW}}-1}b_{-3-j,k}\\
&\quad+\sum_{k=0}^{L_{\mathrm{STW}}-1}b_{-1,k}+\sum_{k=0}^{L_{\mathrm{OSW}}-1}b_{-2,k}\\
b_{0,0} &= 1\Bigg/\left(\frac{CW_{\min}(1-(2P_{coll})^m)}{2(1-2P_{coll})(1-Pf)}+\frac{CW_{\min}((2P_{coll})^m)+1}{2(1-P_{coll})(1-Pf)}\right.\\
&\quad\left.+\frac{L_{\mathrm{CTW}}P_{coll}}{1-P_{coll}}+L_{\mathrm{STW}}+L_{\mathrm{OSW}}\right)
\end{aligned} \tag{2}
$$

Using $b'_{0,0}$ and $b_{0,0}$, we can simply derive the probabilities that a node is transmitting, i.e., τ_W for WiFi and τ_B and BoX-MAC, as follows:

$$\tau_B=3L_{\mathrm{TXB}}\sum_{j=0}^{1}(1-\alpha)b'_{j,-1}=3L_{\mathrm{TXB}}b'_{0,0} \tag{3}$$

$$\tau_W=L_{\mathrm{STW}}\sum_{j=0}^{m}b_{j,0}=\frac{L_{\mathrm{STW}}b_{0,0}}{1-P_{coll}} \tag{4}$$

Knowing τ_B and τ_W, we can calculate the conditional collision probability P_{coll}. For a collision to occur, besides the WiFi device transmitting, there is at least one other device transmitting:

$$P_{coll}=1-(1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B} \tag{5}$$

where N_W and N_B are the number of WiFi and BoX-MAC nodes, respectively.

We remark that the channel busy probabilities, i.e., α and P_f for BoX-MAC and WiFi, respectively, have not been computed yet. The challenge in computing them comes from the fact that the two protocols are extremely different. The main observation that we make is that the channel witnesses all activities of nodes. Consequently, our main idea is to develop a Markov Chain based channel model for symmetric wireless coexistence, the first of its kind.

D. Markov Chain Based Channel Model for Coexistence

We now present the Markov chain based channel model for coexistence. With this model we aim to compute the steady state transition probabilities and stationary distribution of the channel states. Without a Markov Chain model, α and P_f (as computed from the 802.11 and BoX-MAC Markov Chains) cannot be proven to reflect the steady state transition probabilities of the channel. Thus, a Markov Chain model is expected to be more accurate. Evidence was given in [13], where a channel model improved accuracy in P_f computation for 802.11 wireless networks. Additionally, our Markov Chain based channel model simplifies the analysis for α and P_f computation, which we expect to be extremely beneficial when heterogeneous coexistent networks will be considered (i.e., different nodes have different contention windows).

We note that α and P_f are conditional probabilities that depend on nodes "sensing" the channel. From the perspective

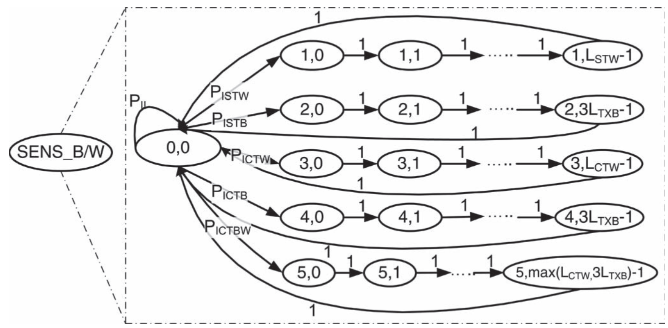

Fig. 4. Markov Chain describing the channel.

of a sensing node, which we call "Tagged Node," the state of the channel (e.g., busy with successful transmission, busy with collision, or idle) and the states of other nodes are interrelated. For a Tagged Node (note: the tagged node simply senses the channel), the channel is busy when any other node transmits; a transmission is successful if only one other node transmits; in all other cases, the channel is idle. Since in our coexistence problem we have devices of different types, the "Tagged Node" can be either a BoX-MAC node or an 802.11 node.

Each channel sensing process, i.e., SENS_B and SENS_W for BoX-MAC and WiFi, respectively, is modeled as a Markov Chain as shown in Fig. 4. Let $v(t)$ and $x(t)$ be the two stochastic processes representing the state of the channel and a counter process, respectively. $v(t) = 0$ represents the state where the channel is idle; $v(t) = 1$ represents the state where the channel is busy with a successful 802.11 transmission; $v(t) = 2$ represents the state where the channel is busy with a successful BoX-MAC transmission; $v(t) = 3$ represents the channel busy with two or more 802.11 nodes transmitting, which leads to a collision; $v(t) = 4$ represents the channel busy with two or more BoX-MAC nodes transmissions, which leads to a collision; and $v(t) = 5$ represents the channel busy with at least one 802.11 node and at least one BoX-MAC transmitting, which leads to a collision. We divide the busy states of the channel in this manner because the duration of each transmission is different (depending on the types of transmitters, i.e., BoX-MAC or WiFi), but deterministic (note that we assume that devices of the same type transmit packets of same length). From each busy state, the channel returns to an idle state after the transmission. We note that if the transmission was not deterministic (i.e., packet length of same type devices can be different), the model would have a single state for the busy channel, that returns to the idle state *with some probability*. This would make the analysis much more difficult and we leave it for future work. The single step transition probabilities are as follows:

$$\Pr[0,0|0,0] = P_{\mathrm{II}}, \qquad \Pr[1,0|0,0] = P_{\mathrm{ISTW}}$$
$$\Pr[2,0|0,0] = P_{\mathrm{ITSB}}, \quad \Pr[3,0|0,0] = P_{\mathrm{ICTW}}$$
$$\Pr[4,0|0,0] = P_{\mathrm{ICTB}}, \quad \Pr[5,0|0,0] = P_{\mathrm{ICTBW}}$$
$$\Pr[i,j|i,j-1] = 1, \quad \text{for } i = 2,\ldots,5,\ j = 1,\ldots,L_i - 1$$
$$\Pr[0,0|i,L_i - 1] = 1, \quad \text{for } i = 1,\ldots,5$$

where $L_1 = L_{\mathrm{STW}}$, $L_2 = 3L_{\mathrm{TXB}}$, $L_3 = L_{\mathrm{CTW}}$, $L_4 = 3L_{\mathrm{TXB}}$, and $L_5 = \max(L_{\mathrm{CTW}}, 3L_{\mathrm{TXB}})$.

As mentioned, when considering the channel state, there must be a reference node, i.e., the node that is sensing the channel or the *Tagged Node*. Since there are two different types of Tagged Nodes (802.11 and BoX-MAC), two different analyses of the channel are needed.

First, if a BoX-MAC node is the Tagged Node, the probability that it finds the channel idle, namely P'_{II}, is the probability that none of the nodes other than the BoX-MAC node is transmitting. From the idle state, the probability that the channel will contain a successful 802.11 transmission in the next step P'_{ISTW} is the probability that one of the WiFi nodes is transmitting while no other BoX-MAC node is transmitting. Similarly, the probability that the channel goes from an idle state to a state of successful BoX-MAC transmission P'_{ISTB} is the probability that exactly one of the remaining BoX-MAC nodes is transmitting. Collisions are predicted based on the probability that two or more nodes will enter a transmission state simultaneously (P'_{ICTW}, P'_{ICTB} and P'_{ICTBW} for WiFi devices, for BoX-MAC devices, and for WiFi and BoX-MAC devices, respectively). A node l has a probability of transmission τ_l that is determined by its contention window size. The resulting single step transition probabilities of the channel Markov Chain are:

$$P'_{\mathrm{II}} = (1-\tau_W)^{N_W}(1-\tau_B)^{N_B-1}$$
$$P'_{\mathrm{ISTW}} = N_W\tau_W(1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B-1} \tag{6}$$
$$P'_{\mathrm{ISTB}} = N_B\tau_B(1-\tau_B)^{N_B-2}(1-\tau_W)^{N_W} \tag{7}$$
$$P'_{\mathrm{ICTW}} = (1-\tau_B)^{N_B-1}\left(1 - N_W\tau_W(1-\tau_W)^{N_W-1} - (1-\tau_W)^{N_W}\right) \tag{8}$$
$$P'_{\mathrm{ICTB}} = (1-\tau_W)^{N_W}(1 - N_B\tau_B(1-\tau_B)^{N_B-2} - (1-\tau_B)^{N_B-1}) \tag{9}$$
$$P'_{\mathrm{ICTBW}} = \left(1-(1-\tau_W)^{N_W}\right)\left(1-(1-\tau_B)^{N_B-1}\right) \tag{10}$$

If the Tagged Node is an 802.11 node: from the idle state, the probability that the channel will contain a successful WiFi transmission in the next step is the probability that one of the remaining WiFi nodes is transmitting while none of BoX-MAC nodes is transmitting. To keep the text concise, we omit the descriptions of other probabilities. The resulting single step transition probabilities are as follows:

$$P_{\mathrm{II}} = (1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B}$$
$$P_{\mathrm{ISTW}} = N_W\tau_W(1-\tau_W)^{N_W-2}(1-\tau_B)^{N_B} \tag{11}$$
$$P_{\mathrm{ISTB}} = N_B\tau_B(1-\tau_B)^{N_B-1}(1-\tau_W)^{N_W-1} \tag{12}$$
$$P_{\mathrm{ICTW}} = (1-\tau_B)^{N_B}\left(1 - N_W\tau_W(1-\tau_W)^{N_W-2} - (1-\tau_W)^{N_W-1}\right) \tag{13}$$
$$P_{\mathrm{ICTB}} = (1-\tau_W)^{N_W-1}\left(1 - N_B\tau_B(1-\tau_B)^{N_B-1} - (1-\tau_B)^{N_B}\right) \tag{14}$$
$$P_{\mathrm{ICTBW}} = \left(1-(1-\tau_W)^{N_W-1}\right)\left(1-(1-\tau_B)^{N_B}\right) \tag{15}$$

Given the single step transition probabilities, we derive the stationary distributions of this Markov Chain as follows (for the Tagged Node being BoX-MAC):

$$v'_{i,j} = v'_{i,0} \text{ for } i \in \{1, 2, \ldots, 5\}, \quad v'_{1,0} = P'_{\mathrm{ISTW}} v'_{0,0}$$
$$v'_{2,0} = P'_{\mathrm{ISTB}} v_{0,0}, \qquad v'_{3,0} = P'_{\mathrm{ICTW}} v_{0,0}$$
$$v'_{4,0} = P'_{\mathrm{ICTB}} v_{0,0}, \qquad v'_{5,0} = P'_{\mathrm{ICTBW}} v_{0,0}$$

where $v'_{0,0}$ is steady probability that the tagged BoX-MAC node senses the channel idle.

The normalization condition for the channel model yields the following:

$$v'_{0,0} + \sum_{j=0}^{L_{\mathrm{STW}}-1} v'_{1,j} + \sum_{j=0}^{3L_{\mathrm{TXB}}-1} v'_{2,j} + \sum_{j=0}^{L_{\mathrm{CTW}}-1} v'_{3,j} + \sum_{j=0}^{3L_{\mathrm{TXB}}-1} v'_{4,j} + \sum_{j=0}^{\max(L_{\mathrm{CTW}},3L_{\mathrm{TXB}})-1} v'_{5,j} = 1$$

From the above equation, we obtain:

$$v'_{0,0} = 1/\left(1 + L_{\mathrm{STW}} P'_{\mathrm{ISTW}} + 3L_{\mathrm{TXB}} P'_{\mathrm{ISTB}} + L_{\mathrm{CTW}} P'_{\mathrm{ICTW}} + 3L_{\mathrm{TXB}} P'_{\mathrm{ICTB}} + \max(L_{\mathrm{CTW}}, 3L_{\mathrm{TXB}}) P'_{\mathrm{ICTBW}}\right) \tag{16}$$

where, from here onwards, the variables in the form of L_{xxx} represent the corresponding duration when event xxx happens.

Using a similar derivation, for an 802.11 node being the Tagged Node, we obtain:

$$v_{0,0} = 1/\left(1 + L_{\mathrm{STW}} P_{\mathrm{ISTW}} + 3L_{\mathrm{TXB}} P_{\mathrm{ISTB}} + L_{\mathrm{CTW}} P_{\mathrm{ICTW}} + 3L_{\mathrm{TXB}} P_{\mathrm{ICTB}} + \max(L_{\mathrm{CTW}}, 3L_{\mathrm{TXB}}) P_{\mathrm{ICTBW}}\right) \tag{17}$$

where $v_{0,0}$ is steady probability that the Tagged Node senses the channel idle.

We now derive α and P_f based on the channel model. α and P_f are the conditional probabilities that the channel has a transmission. Therefore, each is the sum of probabilities that the channel is in any state other than the idle state. Consider α as observed by a BoX-MAC node l sensing the channel when the backoff counter is 0. Now the BoX-MAC node is the Tagged Node, i.e., the channel as observed by the Tagged Node contains all N_W 802.11 nodes and $N_B \setminus \{l\}$ BoX-MAC nodes. Similarly, P_f represents the probability that the channel is not in an idle state for an 802.11 Tagged Node s, i.e., $N_W \setminus \{s\}$ WiFi nodes and N_B BoX-MAC nodes. Therefore, α and P_f can be expressed as follows:

$$\alpha = 1 - v'_{0,0} \tag{18}$$

$$P_f = 1 - v_{0,0} \tag{19}$$

E. Throughput Analysis for Coexisting 802.11 and BoX-MAC

With the help of stationary distribution and normalization conditions, we have successfully derived the variables that reflect the state of the channel, namely α and P_f. These can be determined by numerically solving a set of nonlinear equations. Several performance metrics can be derived from these probabilities. In this paper, we are interested in the saturation aggregate throughput.

The normalized saturation throughput of BoX-MAC is the fraction of time that the channel is busy with a successful BoX-MAC transmission, given by:

$$S_{\mathrm{BoX-MAC}} = 3Lp_{\mathrm{TXB}} P_{\mathrm{STB}} /(P_{\mathrm{I}} + 3L_{\mathrm{TXB}} P_{\mathrm{STB}} + 3L_{\mathrm{TXB}} P_{\mathrm{CTB}} + L_{\mathrm{STW}} P_{\mathrm{STW}} + L_{\mathrm{CTW}} P_{\mathrm{CTW}} + 3L_{\mathrm{TXB}} P_{\mathrm{CTBW}}) \tag{20}$$

where Lp_{TXB} is actual packet size and P_{I} is the probability that the channel is in idle state. P_{STB} and P_{STW} are probabilities that a successful transmission occurs, for BoX-MAC and WiFi, respectively. Likewise, P_{CTB} and P_{CTW} are collision probabilities among pure BoX-MAC and 802.11, respectively, while P_{CTBW} indicates the probability of collision caused by simultaneous transmission of BoX-MAC and 802.11. The variables in the form of L_{xxx} represent the corresponding duration when event xxx happens. The expression of the state probabilities mentioned above are as follows.

$$P_{\mathrm{I}} = (1-\tau_W)^{N_W}(1-\tau_B)^{N_B}$$
$$P_{\mathrm{STW}} = N_W \tau_W (1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B} \tag{21}$$
$$P_{\mathrm{STB}} = N_B \tau_B (1-\tau_B)^{N_B-1}(1-\tau_W)^{N_W} \tag{22}$$
$$P_{\mathrm{CTW}} = (1-\tau_B)^{N_B}\left(1 - N_W\tau_W(1-\tau_W)^{N_W-1} - (1-\tau_W)^{N_W}\right) \tag{23}$$
$$P_{\mathrm{CTB}} = (1-\tau_W)^{N_W}\left(1 - N_B\tau_B(1-\tau_B)^{N_B-1} - (1-\tau_B)^{N_B}\right) \tag{24}$$
$$P_{\mathrm{CTBW}} = \left(1-(1-\tau_W)^{N_W}\right)\left(1-(1-\tau_B)^{N_B}\right) \tag{25}$$

Notably, these expressions are quite similar to those for the transitional probabilities P_{II}, P_{STW}, etc. as shown in (6)–(15); however, the equations here are describing the channel state probabilities while the earlier ones were for the channel model (which depend on an observation node). The normalized saturation throughput of 802.11 is the fraction of time that the channel is busy with a successful 802.11 transmission given by:

$$S_{802.11} = Lp_{\mathrm{STW}} P_{\mathrm{STW}}/(P_{\mathrm{I}} + 3L_{\mathrm{TXB}} P_{\mathrm{STB}} + 3L_{\mathrm{TXB}} P_{\mathrm{CTB}} + L_{\mathrm{STW}} P_{\mathrm{STW}} + L_{\mathrm{CTW}} P_{\mathrm{CTW}} + 3L_{\mathrm{TXB}} P_{\mathrm{CTBW}}) \tag{26}$$

where Lp_{STW} is the actual packet size.

We derive expressions for τ_W and τ_B from their respective Markov Chain models as functions of the contention window sizes and channel states, i.e., $CW_{\min}$, $CW_{\max}$ and P_f for

802.11 nodes, and CW_{init}, CW_{cong} and α for BoX-MAC nodes as:

$$\tau_W = \frac{L_{\text{STW}}}{(1-P_{coll})} \cdot 1 \bigg/ \left(\frac{CW_{\min}\left(1-(2P_{coll})^m\right)}{2(1-2P_{coll})(1-Pf)} + \frac{CW_{\min}\left((2P_{coll})^m\right)+1}{2(1-P_{coll})(1-Pf)} + \frac{L_{\text{CTW}}P_{coll}}{1-P_{coll}} + L_{\text{STW}} + L_{\text{OSW}} \right) \tag{27}$$

$$\tau_B = \frac{L_{\text{TXB}}}{\left(\frac{W_0'+1}{2} + \frac{(W_1'+1)x}{2(1-x)} + \frac{1-\alpha}{1-x} + L_{\text{TXB}} + L_{\text{OSB}}\right)} \tag{28}$$

These can be used to tune protocol parameters based on expected τ_l for a node l.

IV. Contention Window Tuning for QoS and Fairness

In this section we present Contention Window (CW) tuning mechanisms intended to control QoS as well as fairness by changing the CW size on individual nodes. CW is critical for all contention based protocols because it directly controls the transmission probability, thus impacting the throughput [16]–[19]. CW tuning is the main objective of most state of the art optimization protocols, where an accurate model plays the key role. Since we have proposed the first Markov Chain based model for coexistence, the capability to tune the CW is important. As shown by literature and confirmed by our experimental results [15], the congested CW size of BoX-MAC (CW_{cong}) and the minimum CW size of 802.11 ($CW_{\min}$) have a significant impact on the throughput achieved by individual nodes in a coexisting network, while the initial CW size of BoX-MAC (CW_{init}) and the maximal CW size of 802.11 ($CW_{\max}$) do not. Thus for simplicity, we only consider tuning CW_{cong} and $CW_{\min}$, and treat CW_{init} and $CW_{\max}$ as fixed.

The Markov Chain model presented in the previous section provides a method to estimate the saturation throughput of coexisting networks of 802.11 and BoX-MAC nodes. The model also provides a mechanism to estimate the probability of transmission τ_B or τ_W of a node, given the CW sizes, the packet size, and an observed status of the channel, i.e., α as observed by a BoX-MAC node, and P_{coll} and P_f as observed by a 802.11 node.

A. Centralized CW Tuning Method for QoS

QoS is extremely important when heterogeneous protocols compete for the same medium. For instance, low bit rate protocols (such as ZigBee and BoX-MAC) cannot easily capture the channel due to their non-aggressive nature. On the other hand, these protocols can severely degrade high bit rate protocols (like 802.11) when they capture the channel (because of the former's low transmission rate). Our CW tuning method helps mitigate these effects.

It is problematic to claim that the QoS is the ratio of the two throughputs because of the order of magnitude difference in PHY bit rates. Therefore, we define the QoS metric under coexistence as the ratio of the successful transmission probabilities of 802.11 and BoX-MAC [27]. The probability that a transmission is successful is expressed as:

$$SU_B = \tau_B(1-\tau_B)^{N_B-1}(1-\tau_W)^{N_W}$$
$$SU_W = \tau_W(1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B}$$

Hence, the QoS metric, denoted by ϕ, can be written as:

$$QoS = \phi = \frac{SU_B}{SU_W} = \frac{\tau_B(1-\tau_B)^{N_B-1}(1-\tau_W)^{N_W}}{\tau_W(1-\tau_W)^{N_W-1}(1-\tau_B)^{N_B}} = \frac{\tau_B(1-\tau_W)}{\tau_W(1-\tau_B)}$$

We then obtain an expression of τ_W in terms of τ_B and ϕ:

$$\tau_W = \frac{\tau_B}{\frac{1-\tau_B}{\phi} + \tau_B} \tag{29}$$

There is a multitude of τ_B and τ_W combinations that satisfy ϕ. In this paper, we aim to maximize the total throughput of a given network. Since the aggregate throughput, i.e., $S_{802.11}$ and $S_{\text{BoX-MAC}}$, depends on τ_B and τ_W, we maximize the total throughput as $S_{\text{total}} = S_{\text{BoX-MAC}} + S_{802.11}$, while maintaining the QoS requirement.

Based on (20) and (26), we obtain the expression for $S_{\text{BoX-MAC}} + S_{802.11}$ as:

$$\begin{aligned} S_{\text{total}} &= S_{\text{BoX-MAC}} + S_{802.11} \\ &= (3Lp_{\text{TXB}}P_{\text{STB}} + Lp_{\text{STW}}P_{\text{STW}}) \\ &\quad /(P_{\text{I}} + 3L_{\text{TXB}}P_{\text{STB}} + 3L_{\text{TXB}}P_{\text{CTB}} + L_{\text{STW}}P_{\text{STW}} \\ &\quad + L_{\text{CTW}}P_{\text{CTW}} + 3L_{\text{TXB}}P_{\text{CTBW}}) \end{aligned} \tag{30}$$

All P_{xxx} above are functions of τ_B and τ_W. Using (29), we substitute all τ_W with τ_B, and express S_{total} as a function of τ_B only. Note that the QoS ratio ϕ is user specified. In order to get the maximum value for S_{total}, we take the derivative of (30) w.r.t τ_B, and set it to 0, i.e., $S'_{\text{total}} = 0$. Because of the computational complexity of solving this equation, we use an approximation method. Under the condition $\tau_B \ll 1$, the following approximation holds:

$$(1-\tau_B)^n \approx 1 - n\tau_B + \frac{n(n-1)}{2}\tau_B^2 \tag{31}$$

We can thus significantly simplify the expression for S_{total}, reducing the complexity of solving (30) and making it feasible to run on COTS computer hardware, especially when the number of nodes is large. By numerically solving (31), we obtain the value of τ_B and subsequently, τ_W through (29). A few more unknowns need to be computed before CW sizes can be obtained. α, P_{coll} and P_f are computed using (5) and (6)–(19). Finally, by substituting τ_B, τ_W, P_{coll}, α and P_f into (27), (28) and treating all other variables as constants, we obtain the contention window sizes for BoX-MAC and 802.11.

B. Distributed CW Tuning Method for Fairness

As described before, $CW_{\min}$ and CW_{cong} are critical for coexisting wireless networks. Nodes with different $CW_{\min}$ and CW_{cong} achieve different throughput, thereby leading to unfair utilization of the bandwidth. We call this the *fairness problem*. When the nodes of a network are allowed to tune their parameters themselves, it becomes important to show the existence of an equilibrium point from which no node has the incentive to modify its parameters [25].

To solve the fairness problem, we propose a game theoretic approach similar to the one presented by Jin and Kesidis [25]. We define a concave maximization function of τ_i (transmission probability of node i) for each node as $\max(U_i - D_i)$, where U_i is the utility, and D_i is the disutility, or cost experienced by the node n_i for a given τ_i and α.

$$U_i - D_i = \left(\frac{2(N_W + N_B - 1)log(1+\tau_i)}{L}\right) - \frac{\tau_i \alpha}{L(1+\alpha)}$$

where $L = L_{\text{STW}}$ is for 802.11 nodes and L_{TXB} is for BoX-MAC nodes. Here, $\tau_i(\alpha/1+\alpha)$ is an approximation for the expected collision probability.

In this CW tuning method, each node always tries to maximize $U_i - D_i$. This can be thought of as a selfish behavior. The game model is such that each node decides $\tau = [\tau_1, \tau_2, \ldots, \tau_{N_W}, \tau_{N_W+1} \ldots \tau_{N_W+N_B}]'$ to maximize $f(\tau) = [U_1 - D_1, U_2 - D_2, \ldots, U_{N_W+N_B} - D_{N_w+N_B}]'$. For a node n_i, $U_i - D_i = ((2(N_W + N_B - 1)log(1+\tau_i)/L)) - (\tau_i\alpha_i/L(1+\alpha_i))$. The gradient, $\nabla f(\tau)_i = ((2(N_W + N_B - 1)/L(1+\tau_i)) - (\alpha_i/1+\alpha_i))$. To prove that $f(\tau)$ is concave, we show that the Jacobian of $\nabla f(\tau)$, $F(\tau)$ is negative definite, i.e., has only negative eigenvalues.

A diagonal element of F, $F_{ii} = (-2(N_W + N + B - 1)/L(1+\tau_i)^2)$, and non-diagonal element $F_{ij} = (-1/L)((1-\alpha_j)^2/(1+\alpha_j)^2)\Pi_{k\neq i\neq j}(1-\tau_k)$. Since $((1-\alpha_j)^2/(1+\alpha_j)^2) \leq 1$ and $\Pi_{k\neq i\neq j}(1-\tau_k) < 1$, $|F_{ij}| < (1/L)$. Therefore, $\sum_{j\neq i}|F_{ij}| < (N_W + N_B - 1/L)$. Since τ is the probability that a node attempts to begin a transmission, it is always less than 0.414, $(1+\tau_i)^2 < 2$, i.e., $(1/(1+\tau_i)^2) > (1/2)$, which proves that $|F_{ii}| < \sum_{j\neq i}|F_{ij}|$ for all i.

Since F_{ii} is negative, and by the Gerschgorin circle theorem, all eigenvalues lie in circles with center at F_{ii} and radius $\sum_{j\neq i}|F_{ij}|$, F is negative definite and this proves that the game has a concave objective. As proven by Rosen [28], an n-player non-cooperative game where each player is selfish and maximizes a concave objective reaches Nash Equilibrium. Hence, there exists a Nash equilibrium, and therefore, our proposed mechanism will reach a stable state where no node benefits from changing their parameters, even though each node behaves selfishly (given that the other nodes do not change their parameters). We also observe that at the Nash equilibrium, nodes of the same kind choose the same parameters.

Rosen also proposed a gradient projection method to iteratively reach Nash Equilibrium. We use this result to define a gradient projection iteration to reach Nash Equilibrium:

$$\tau_{k+1} = \tau_k + \frac{\epsilon}{L}\left(\frac{2(N_W + N_B - 1)}{1+\tau_k} - \frac{\alpha_k}{1+\alpha_k}\right)$$

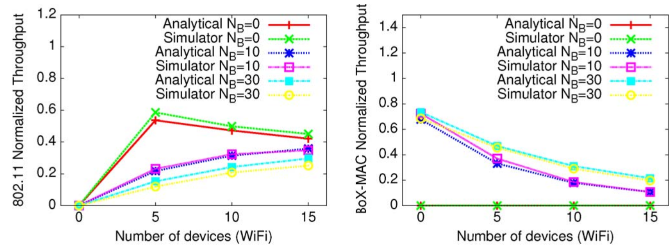

Fig. 5. Throughput of model and simulator versus number of devices.

where ϵ is the step size of the gradient projection method, τ_k is the τ observed by the node in the kth step and α_k is the α observed by the node in the kth step.

As we know from the Markov chain model, τ is a function of the contention window sizes. We can therefore solve for $CW_{\min}$ for 802.11 and CW_{cong} for BoX-MAC at the $(k+1)$th step from τ_{k+1}. For a sufficiently large k, the system reaches a stable point, i.e. Nash equilibrium, as proven by Rosen. Since the CW sizes have to be chosen from a set (combinatorial), the CW sizes that result in a transmission probability closest to τ_{k+1} are chosen at each step. We have shown that the fairness problem can be solved using this method, by proving that the individual throughput of nodes of the same type are equal at Nash Equilibrium.

V. Performance Evaluation

In this section we investigate the accuracy of our analytical model and present the performance evaluation of our CW tuning methods. For this, we use the coexistence simulator presented in [15]. We chose to use our own simulator because no existing academic or commercial simulator, e.g., ns2, QualNet, etc., supports wireless coexistence scenarios (please note, coexistence of different wireless MAC protocols). Additionally, we need the ability to control all low level parameters in the simulator (e.g., computations of different collision probabilities), which is a capability typically not exposed to users, in commercial simulators. The Monte Carlo based simulator in [15], mimics 802.11 and 802.15.4 protocols at the MAC layer. The simulator's accuracy has been validated through extensive experiments on real hardware (4 MikroTik WiFi routers, 8 TelosB motes and over 60 million transmitted packets). We chose a simulator based approach for validating our analytical model and evaluating our CW tuning methods because it is rather very difficult to obtain results from large scale deployments (i.e., scalability issue), and still maintain the ability to evaluate multiple configuration settings (i.e., various protocol parameters).

Simulation Parameters: Since 802.11 DCF and BoX-MAC are CSMA/CA-based, the *contention window size* is a key parameter which impacts the throughput (e.g., a smaller window size is more aggressive, but it gives more opportunities for collisions). The packet size is also an important factor since it impacts the time the channel is occupied. In our evaluations, the metrics we choose for 802.11 DCF are the minimum contention window size $CW_{\min} = \{16, 32, 64\}$, the maximum contention window size $CW_{\max} = \{256, 512, 1024\}$ and the packet size $P_W = \{500, 1000, 1500\}$. Similarly, the metrics for BoX-MAC

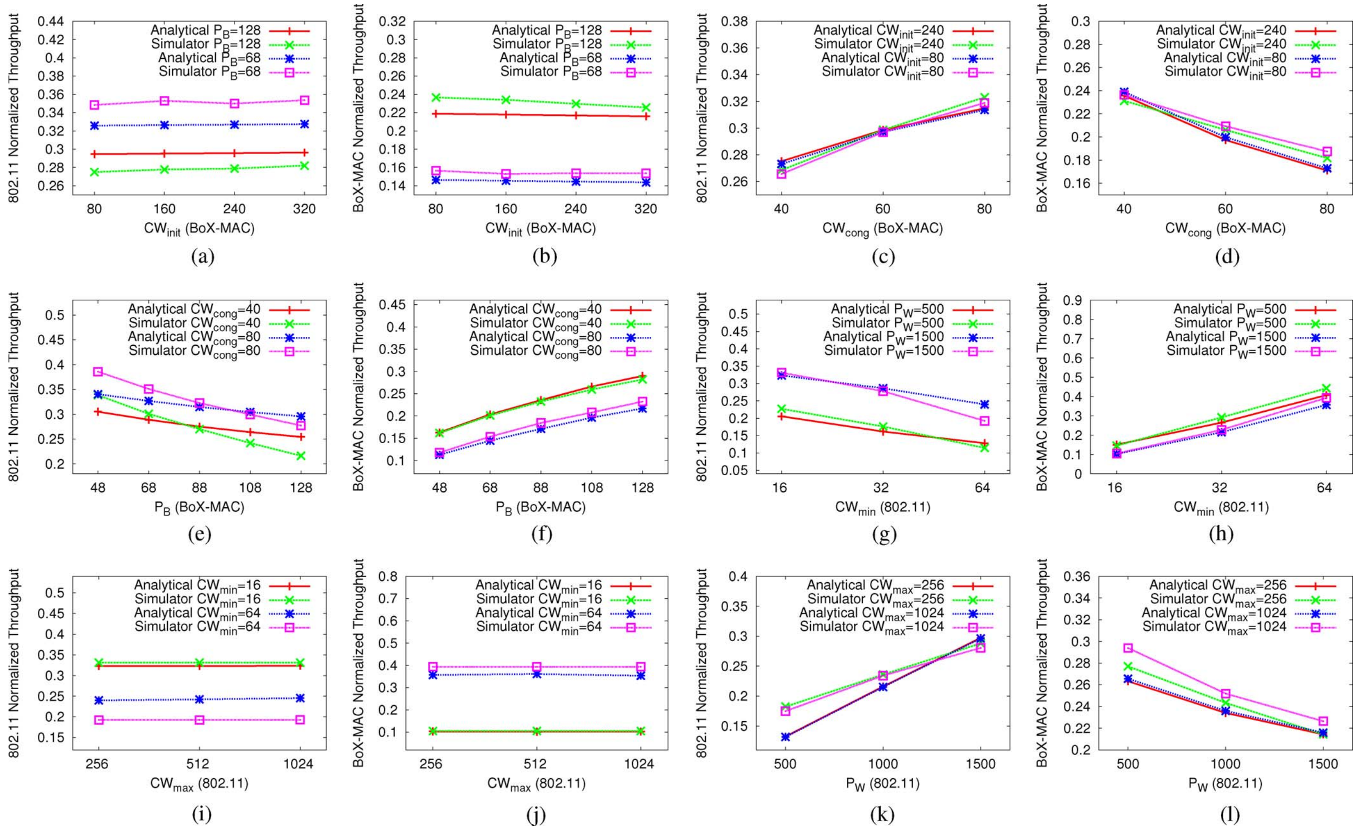

Fig. 6. Model validation: 802.11 and BoX-MAC throughput from model and simulator when varying BoX-MAC parameters [(a)–(b) CW_{init}; (c)–(d) CW_{cong}; and (e)–(f) P_B] and when varying 802.11 parameters [(g)–(h) $CW_{\min}$; (i)–(j) $CW_{\max}$; and (k)–(l) P_W].

are the initial contention window size $CW_{\text{init}} = \{80, 160, 240, 320\}$, congested contention window size $CW_{\text{cong}} = \{40, 60, 80\}$, and the packet size $P_B = \{48, 68, 88, 108, 128\}$. We used $\sum_{i=1}^{k}(|T_{S_i} - T_{H_i}| \times 2/(T_{S_i} + T_{H_i}))/k$, the classical *average difference* between two sets of data (i.e., simulator and analytical model in our case) as the evaluation metric, where k is the number of tests, and T_S and T_H are simulator and model throughput, respectively.

A. Analytical Model Validation

We compare the normalized throughput (as in [13], [20]) obtained from our analytical model with that obtained from the Monte Carlo simulator. The parameters we vary are: the contention window size $CW_{\min}$, maximum contention window size $CW_{\max}$, and the packet size P_W of WiFi, and the initial contention window size CW_{init}, contention window size CW_{cong} and the packet size P_B for BoX-MAC. The default values of these parameters are: $CW_{\min} = 32$, $CW_{\max} = 1024$, $P_W = 1500$ for WiFi, and $CW_{\text{init}} = 320$, $CW_{\text{cong}} = 80$, $P_B = 128$. Due to approximations made in the Markov Chain model (e.g., DIFS delay after backoff freeze), we see differences in results obtained from simulator and the analytical model. We use the aforementioned average difference metric to compare them.

Scaling the Number of Devices: Analyzing the coexistence of WiFi and WSN by varying the number of devices in a real implementation is tedious and time consuming. The same can be done by merely varying these parameters in the analytical model and simulator. Fig. 5 depicts the comparison of normalized throughput when the number of WiFi and BoX-MAC devices are varied, setting other parameters to their default values. The results indicate that increasing the number of WiFi devices increases the throughput of WiFi and degrades the throughput of BoX-MAC, while increasing the number of BoX-MAC devices increases BoX-MAC throughput and degrades WiFi throughput. Interestingly, there is good agreement between simulator and analysis with an average difference of 3% and with worst difference of 6%.

TABLE II
SETTINGS FOR QoS TUNING EVALUATION

No. of BoX-MAC	No. of WiFi	$\phi1$	$\phi2$	$\phi3$
5	5	0.1	1	10
10	10	0.1	1	10
20	20	0.1	1	10
5	10	0.1	1	10
10	20	0.1	1	10
10	5	0.1	1	10
20	10	0.1	1	10

Throughput Comparison: To analyze the impact of BoX-MAC parameters on throughput, as obtained from the simulator and from the analytical model, we considered a scenario with 15 WiFi, 30 BoX-MAC devices and default WiFi parameters. The results are depicted in Fig. 6. Fig. 6(a) and (b) show that increasing CW_{init} the throughput of BoX-MAC remains almost the same. This is because with a big number of devices, the probability of transmitting after the first backoff attempt is low. As CW_{cong} increases [in Fig. 6(c) and (d)], the backoff overhead of BoX-MAC increases, thus making its throughput

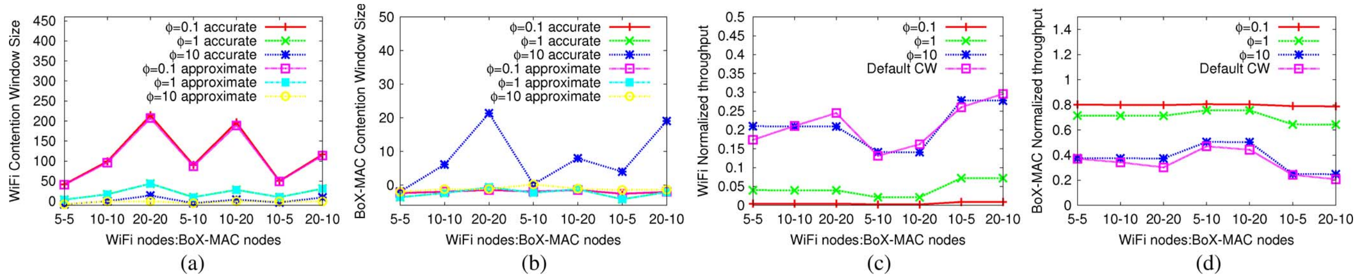

Fig. 7. **CW tuning for QoS**: Optimized CW size for (a) 802.11 and (b) BoX-MAC nodes based on deployment size. The realized throughput for these CW sizes is shown in (c) for WiFi and (d) for BoX-MAC.

decrease. The trend for varying P_B is expected [in Fig. 6(e) and (f)], since a big packet size implies larger payload and longer channel occupancy, both of which benefit BoX-MAC. Results show an excellent agreement (average difference of 3% and worst case of 6%) between analysis and simulator. Notably, since CW_{init} is not effective in throughput scaling, it should not be considered as a parameter for tuning.

The impact of WiFi parameters on throughput derived from simulator and analytical model was analyzed with the same number of devices and default BoX-MAC parameters. Results are depicted in Fig. 6. One can observe that $CW_{\min}$ has a significant impact on throughput, as shown in Fig. 6(g) and (h). In terms of $CW_{\max}$, since the collision probability is not too high with this number of nodes, it is unlikely that WiFi nodes reach the maximum contention window size, for successful transmissions. Also, $CW_{\max}$ does not affect the throughput, as shown in Fig. 6(i) and (j). As for P_W (in Fig. 6(k) and (l)), the trend is also expected, i.e., it impacts the throughput greatly. These results also show a remarkable agreement (average difference of 2% with a worst case of 5%) between analysis and simulator. The agreement between the simulator and the analysis in all these experiments validates the analytical model as an extremely valuable tool. Similarly, $CW_{\max}$ is not an effective parameter for throughput tuning.

B. Contention Window Tuning Evaluation

We evaluate the tuning methods proposed in Section IV using our simulator. First, we demonstrate that the QoS can be satisfied through our CW tuning method, and that the total throughput is also maximized. Second, we show that the tuning method we present reaches Nash equilibrium. Then we show that attaining Nash equilibrium can also solve the fairness problem, i.e., unfair bandwidth utilization, which is seen when nodes with different $CW_{\min}$ and CW_{cong} have different throughput.

Evaluation of QoS Tuning: The basic idea of the evaluation is that given the number of BoX-MAC and WiFi nodes, and information like packet size, a centralized master device can communicate with both types of devices and calculate the contention window size based on the CW tuning method. Each node will be informed of the CW size to use, and as a result, the QoS requirement is met while the total throughput is maximized. We perform simulations using several combinations of BoX-MAC and WiFi nodes, and different QoS settings, i.e., ϕ, as in Table II. We now show the optimal contention window sizes that satisfy the requirement.

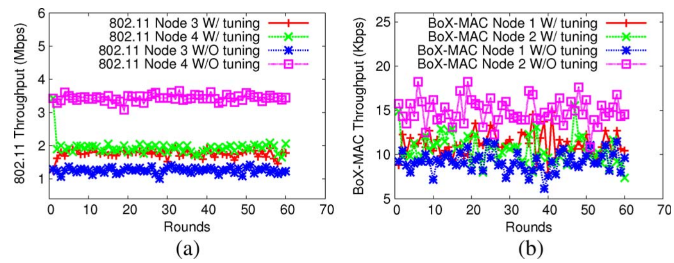

Fig. 8. Throughput of 802.11 and BoX-MAC are fairly shared by tuning CW_{cong} and $CW_{\min}$.

As shown in Fig. 7(a) and (b), the optimized CW size is quite different from the default settings ($CW_{\min} = 32$, $CW_{\max} = 1024$, $P_W = 1500$ for WiFi, and $CW_{\text{init}} = 160$, $CW_{\text{cong}} = 80$, $P_B = 128$ for BoX-MAC), which emphasizes the necessity for CW tuning. The realized throughput at these CW sizes is shown in Fig. 7. Notably, the throughput from default contention window size we chose was very close to the optimal value for the QoS parameter $\phi = 10$. For other CW combinations, the results were not as good as when the value was obtained from our optimization.

Evaluation of Fairness Tuning: As mentioned before, the fairness problem is due to nodes choosing different corresponding contention window sizes, i.e., $CW_{\min}$ and CW_{cong}. However, the solution for this problem is quite straightforward, since, generally, nodes of the same kind tend to choose the same parameters when Nash equilibrium is reached which implies that the realized throughputs are equal (they share the bandwidth fairly). We performed simulations for 5 WiFi and 10 BoX-MAC nodes. The nodes initially chose random CW_{cong} and $CW_{\min}$. To compare the results for fairness, we ran two separate tests, whose results are shown in Fig. 8. It is obvious that when the Nash equilibrium was attained, the devices fairly shared the throughput, while without tuning, the throughput can be quite different, based on the parameters they choose initially.

VI. Conclusion and Future Work

We have presented the first analytical model for predicting saturation throughput in symmetric coexisting 802.11 and 802.15.4 networks. Our analysis is anchored in solid theoretical results based on modeling 802.11 DCF and BoX-MAC as Markov Chains, and a channel model that is able to accurately estimate channel busy probabilities. Additionally, we proposed two contention window tuning methods as applications of our modeling tool, and we show that they not only achieve QoS but also fairness. As future work, we aim to extend our model to asymmetric coexistence scenarios, to account for hidden and exposed terminal problems, and to accommodate variable length packets. We are also interested to study throughput as well as energy consumption under non-saturated traffic patterns.

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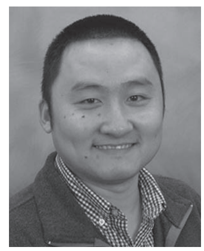

Wei Zhang (S'14) received the B.S. and M.S. degrees in electrical engineering from Northwest Polytechnical University, Xi'an, China in 2005 and 2008, respectively. He is currently working toward the Ph.D. degree in the Department of Computer Science, Texas A&M University, College Station, TX, USA, under the guidance of Dr. R. Stoleru.

His research interests include wireless networks, distributed systems, network modeling, queueing theory, and game theory.

Mahima A. Suresh (S'06) received the B. Tech. degree from the National Institute of Technology, Karnataka, Surathkal, India, in 2009. She is currently working toward the Ph.D. degree in the Department of Computer Science and Engineering, Texas A&M University, College Station, TX, USA.

She is currently a member of the Laboratory for Embedded and Networked Sensor Systems headed by Dr. Radu Stoleru. Her research interests include wireless sensor networks, algorithms and complexity theory, graph theory, and random processes.

Radu Stoleru (M'05) received the Ph.D. degree in computer science from the University of Virginia, Charlottesville, VA, USA in 2007. While at the University of Virginia, Dr. Stoleru received from the Department of Computer Science the Outstanding Graduate Student Research Award for 2007. He is currently an Associate Professor with the Department of Computer Science and Engineering, Texas A&M University, College Station, TX, USA, and the Head of the Laboratory for Embedded and Networked Sensor Systems (LENSS).

His research interests include deeply embedded wireless sensor systems, distributed systems, embedded computing, and computer networking. He is the recipient of the National Science Foundation CAREER Award in 2013. He has authored or coauthored over 70 conference and journal papers with over 2800 citations (Google Scholar). He is currently serving as an editorial board member for three international journals and has served as technical program committee member on numerous international conferences.

Harsha Chenji (S'08) received the B.Tech. degree in electrical and electronics engineering from National Institute of Technology, Karnataka (NITK), Surathkal, India, in 2007 and the M.S. and Ph.D. degrees in computer engineering from Texas A&M University, College Station, TX, USA in 2009 and 2014, respectively, under the supervision of Prof. R. Stoleru. He is currently a Postdoctoral Researcher with the University of Texas at Dallas, Dallas, TX, USA, under Prof. Zygmunt J. Haas. His research interests include quality of service, localization, and energy efficiency in wireless networks.